Lec7 - MA, ARIMAX

Monday, 12 August 2024 15:37

MA = moving average (ממוצע נע)

$b_0 = 1$

$$\hat{y}[n] = \epsilon[n] + b_1\epsilon[n-1] + \cdots + b_q\epsilon[n-q]$$

MA and AR relations

Presenting AR(p) as MA(∞) For AR(1),

AR →

$$\begin{aligned} y[n] &= a_1 y[n-1] + \epsilon[n] \\ &= a_1(a_1 y[n-2] + \epsilon[n-1]) + \epsilon[n] \\ &= a_1^2 y[n-2] + a_1\epsilon[n-1] + \epsilon[n] \\ &= a_1^3 y[n-3] + a_1^2\epsilon[n-2] + a_1\epsilon[n-1] + \epsilon[n] \\ &= \cdots \end{aligned} \tag{20.2}$$

For $a_1 < 1$, $\lim_{k\to\infty} a_1^k \to 0$, therefore

$$y[n] = \epsilon[n] + a_1\epsilon[n-1] + a_1^2\epsilon[n-2] + a_1^3\epsilon[n-3] + \cdots$$

MA →

$$= \sum_{i=1}^{\infty} a_1^j \epsilon[n-j] + \epsilon[n]$$

$y[n-1] = a_1 y[n-2] + \epsilon[n-1]$ — $y[n-j]$

Presenting MA(q) as AR(∞) For MA(1),

MA

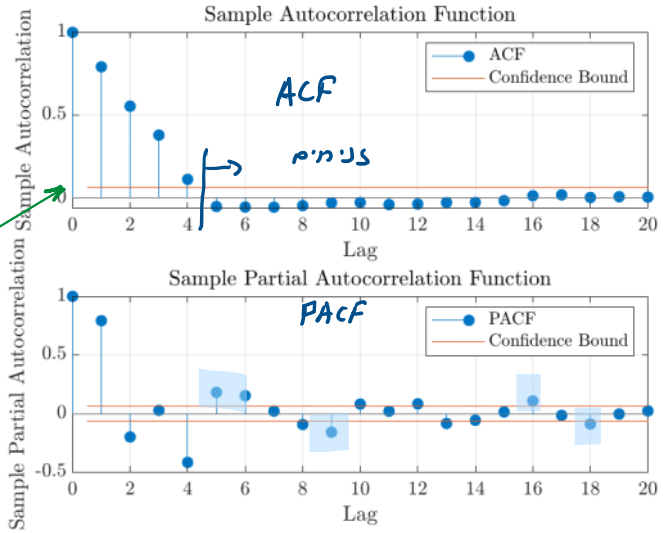
$$\begin{aligned} x[n] &= \epsilon[n] + b_1\epsilon[n-1] \\ x[n-1] &= \epsilon[n-1] + b_1\epsilon[n-2] \\ \epsilon[n-1] &= b_1\epsilon[n-2] - x[n-1] \\ \Rightarrow x[n] &= \epsilon[n] + b_1(b_1\epsilon[n-2] - x[n-1]) \\ &= \epsilon[n] - b_1 x[n-1] + b_1^2\epsilon[n-2] \end{aligned}$$

AR

$$= \sum_{i=0}^{\infty} (-b_1)^i y[n-i]$$

Of cause, $b_1 < 1$ is required.

The relation between MA(q) and ACF



ARMA

$$\begin{aligned} \hat{y}[n] &= a_1 y[n-1] + \cdots + a_p y[n-p] \\ &\quad + b_1\epsilon[n-q] + \cdots + b_q\epsilon[n-q] + \epsilon[n] \\ &= \sum_{i=1}^{p} a_i y[n-i] + \sum_{k=0}^{q} b_k\epsilon[n-k] \end{aligned}$$

AR חלק

MA חלק

$b_0 = 1$,

$+ \mu$ — bias (optional)

ARMAX

The model that combines ARMA (signal history and

ARMAX

The model that combines ARMA (signal history and noise) together with exogenous input (ARX model),

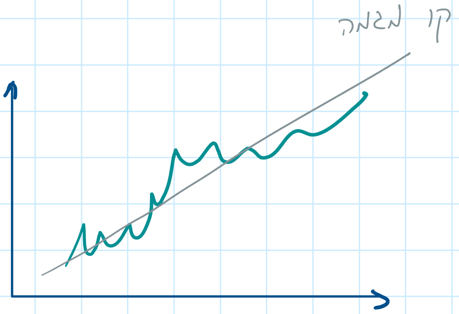
$$\begin{aligned} y[n] &= a_1 y[n-1] + \cdots + a_{n_a} y[n-n_a] \quad AR \\ &+ b_1 x[n-1] + \cdots + b_{n_b} x[n-n_b] \quad X \qquad (20.8) \\ &+ c_1 \epsilon[n-1] + \cdots + c_{n_c} \epsilon[n-n_c] + \epsilon[n] \quad MA \end{aligned}$$

הסטוריה של אות כניסה $x[n]$

"אורך" של המודל

ARI, ARIMA, ARIMAX

Autoregressive integrated moving average



Differencing

שיטה: הפעלת מד' ARMA על האות אחרי הפרש מסדר d

integrated - חזרה לאות מקורי "ע"י אינטגרציה.

הקדמה:

The linear trend is given by

$$x(t) = at + b$$

The basic model with linear trend is

$$y[n] = A + Bn + \epsilon[n]$$

קו ישר פשוט · רעש

De-trending

$$d=1 \quad y'[n] = y[n] - y[n-1] = A + Bn - A - B(n-1) = B$$

קבוע

The quadratic (or parabolic) trend is given by

$$x(t) = at^2 + bt + c$$

$$\begin{aligned} d=2 \quad y''[n] &= y'[n] - y'[n-1] \\ &= y[n] - y[n-1] - (y[n-1] - y[n-2]) \\ &= y[n] - 2y[n-1] + y[n-1] \end{aligned}$$

- The resulting AR model with de-trending is termed ARI(p,d), where d is the difference order (typically $d = 1$ or 2) of the model.
- ARMA model with de-trending is termed ARIMA(p,d,q). ARIMA(p,0,q) is actually ARMA(p,q). A
- ARIMAX model also exists.

($d=0$)

* סיכום: הוספת פרמטר d

* שימוש עבור אותות עם קו מגמה ברור.

Model Selection

ARIMA(p,d,q)

3 מספרים (שלמים)

hyper-parameters - לא נלמדים ישירות

* ניתן לבדוק ב-I - האם יש trend?

* ACF, PACF: שימוש לבחירת p, q

* ACF, PACF : שימוש ס חישוב p , q

ב מספרים (שלמים)

hyper-parameters - לא נלמדים ישירות מהנתונים

* 'parsimony principle' - כמה שפחות עדיף

- כלל אצבע $p+d+q \leq 6$

- שיטות חישוב loss עם penalty (קנס) על מספר פרמטרים