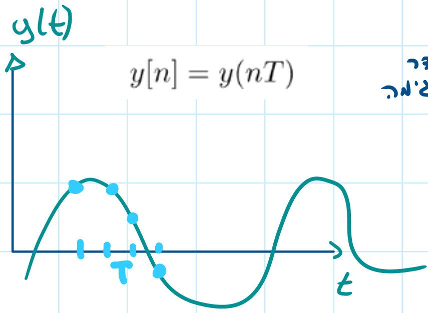
Lec1 - Basic Signal Analysis - Sinusoidal Signal

Sunday, 2 June 2024 18:13



$y[n] = y(nT)$

תדר דגימה $F_s = \frac{1}{T}$ [Hz]

מטרה: ניתוח אות

* $y(t) = \underbrace{A}_{\text{אמפליטודה חיובית}} \cos(2\pi \underbrace{F_0}_{\text{תדר Hz}} t + \underbrace{\theta}_{\text{פאזה}}) + \underbrace{w(t)}_{\text{רעש}}$

$-\pi < \theta \leq \pi$

* $\underbrace{y[n]}_{\text{זמן בדיד}} = y(nT) = A\cos(\underbrace{2\pi F_0 T}_{\omega_0} n + \theta) + w[n]$

הנחה 'יחידה' לגבי הרעש → $\underbrace{E}_{\text{תוחלת/ממוצע}}\{w[n]\} = 0$

$\omega_0 = 2\pi F_0 T = 2\pi \frac{F_0}{F_s}$ [rad]

* **Nyquist criterion** →

$\underbrace{F_0}_{\text{תדר של cos}} < \underbrace{F_s}_{\text{תדר דגימה}}/2$

$\Rightarrow 0 \leq \omega_0 < \pi$

0 ↓ DC, π ↓ $F_s/2$

$x_1(t) = \cos(0.6\pi t), \quad x_2(t) = \cos(2.6\pi t)$

$F_s = 1$ [Hz] ⇒ T = 1sec

$x_1[n] = \cos(0.6\pi n),$

$x_2[n] = \cos(2.6\pi n) = \cos(0.6\pi n + 2\pi n) = x_1[n].$ ← אותם ערכים בזמן בדיד = aliasing

$$\boxed{y[n] = A\cos(\omega_0 n + \theta) + w[n] \quad n = 0, \ldots, L-1}$$

Amplitude and phase

Signal model: The signal is of the form

$$y[n] = A\cos(\omega_0 n + \theta) + \epsilon[n] \quad n = 0, \ldots, L-1 \quad (19.6)$$

- A is positive constant (unknown, what to know)
- $0 \leq \theta < 2\pi$ or $\boxed{-\pi < \theta \leqslant \pi}$ (unknown, whant to know)
- $\epsilon[n]$ is some random noise, $E[\epsilon[n]] = 0$ (zero-mean), no assumptions on a distribution and variance.
- $0 < \omega_0 < \pi$ is known
- L is a number of samples of the signal.

מטרה: "למצוא" A, θ

סיכום: הבעיה בעצם לינארית

$A\cos(\omega_0 n + \theta) = w_c \cos(\omega_0 n) + w_s \sin(\omega_0 n)$

$w_c = A\cos(\theta) \qquad A = \sqrt{w_c^2 + w_c^2}$

$w_s = -A\sin(\theta) \qquad \theta = -\arctan\left(\frac{w_c}{w_s}\right)$

$\hat{y}[n] = w_c \cos(\omega_0 n) + w_s \sin(\omega_0 n)$

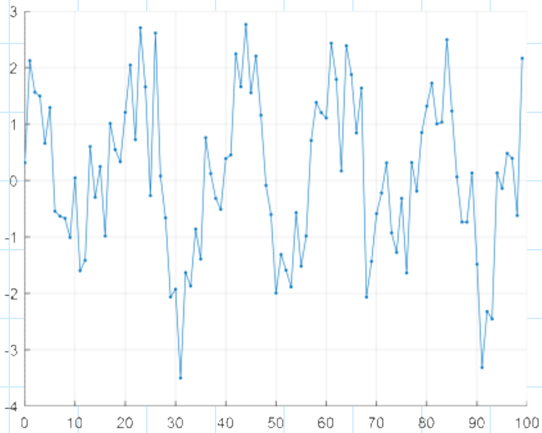
$$\underbrace{\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[L-1] \end{bmatrix}}_{\underline{y}}^{L\times 1} = \underbrace{\begin{bmatrix} \underset{=1}{\cos(\omega_0 \cdot 0)} & \underset{=0}{\sin(\omega_0 \cdot 0)} \\ \cos(\omega_0) & \sin(\omega_0) \\ \vdots & \\ \cos(\omega_0(L-1)) & \sin(\omega_0(L-1)) \end{bmatrix}}_{\underline{X}}^{L\times 2} \underbrace{\begin{bmatrix} w_c \\ w_s \end{bmatrix}}_{\underline{w}}^{2\times 1}$$

$\vdots$

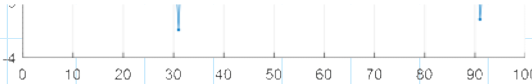
$y[k] = \cos(\omega_0 k) \cdot w_c + \sin(\omega_0 k)\, w_s$

$\vdots$

$$\hat{\mathbf{w}} = \mathbf{X}^{+}\mathbf{y}$$




```
%% dataset
w0 = 0.1*pi;        % שרירותי
A = 1.5;
theta = -pi/4;
L = 100;
n = (0:L-1)';
sigma = 1;          % עוצמת הרעש
y_theory = A*cos(w0*n+theta);
y = y_theory + sigma*randn(L,1);
```



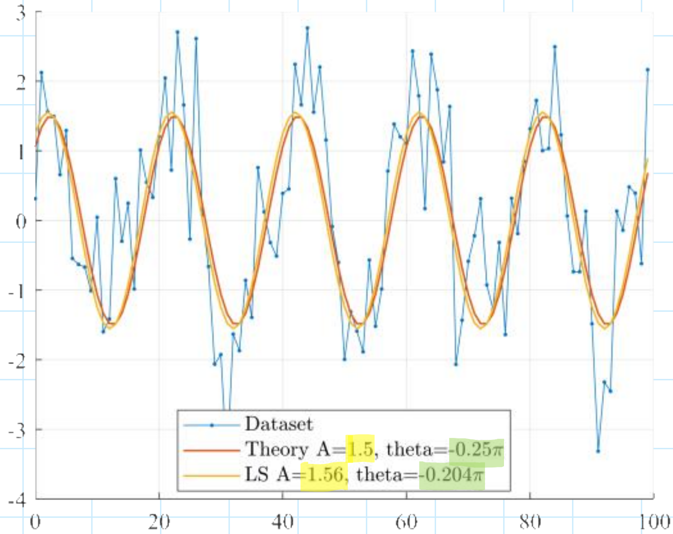

```
%% LS
X = [cos(w0*n) sin(w0*n)];
w_ls = pinv(X)*y;
y_hat = X*w_ls;
%% Amplitude and phase
A_hat = sqrt(sum(w_ls.^2));
theta_hat = -atan2(w_ls(2),w_ls(1));
w_theory = [A*cos(theta); -A*sin(theta)]
```

אות cos "משוחזר"

שונות הרעש

```
sigma = 1;
y_theory = A*cos(w0*n+theta);
y = y_theory + sigma*randn(L,1);
```

תוספת הרעש

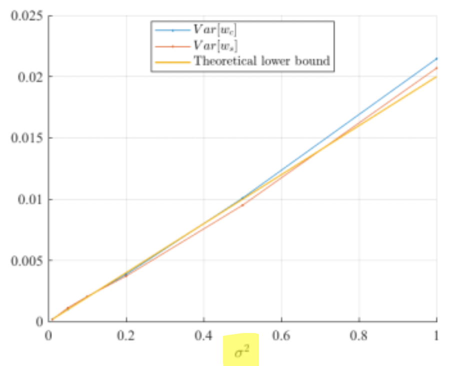


שיעורי הבית

$$\mathrm{Var}\left[\hat{w}_{c,s}\right] \gtrsim \frac{2\sigma^2}{L}$$

Cramer-Rao bound

שונות ההפרש בין תאורטי למחושב = שונות השגיאה



הערה: לבדוק שלא תלוי בערכי: w_0, A, Θ